Calculus Warm Up #1-1

1) Evaluate the limits:

a) $\lim_{x\to 0} \frac{\sin 3x}{x}$

b) $\lim_{x\to\infty} x \sin \frac{1}{x}$

2) An Arithmetic Sequence has a second term and fifth term of - 2 and 7 respectively. Find:

a) the first term

b) a recursive rule for the sequence

c) an explicit rule for the sequence

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1) a) $\lim_{x\to 0} \frac{\sin 3x}{x}$

$= \frac{\sin 0}{0} = \frac{0}{0}$ indeterminate, so use L'Hôpital's Rule

$\lim_{x\to 0} \frac{3\cos 3x}{1} = 3\cos 0 = \boxed{3}$

b) $\lim_{x\to\infty} x \sin \frac{1}{x} = \infty \cdot \sin\left(\frac{1}{\infty}\right) = \infty \cdot 0$ indeterminate

$= \lim_{x\to\infty} \frac{\sin\left(\frac{1}{x}\right)}{\frac{1}{x}}$ ← so Rewrite

$= \frac{0}{\frac{1}{\infty}} \rightarrow \frac{0}{0}$, Now L'Hôpital...

$\lim_{x\to\infty} \frac{-\frac{1}{x^2}\cos\left(\frac{1}{x}\right)}{-\frac{1}{x^2}}$

$= \lim_{x\to\infty} \cos\left(\frac{1}{x}\right) = \cos\left(\frac{1}{\infty}\right) = \boxed{1}$

2) $a_2 = -2, \quad a_5 = 7$

a) $d = \frac{7-(-2)}{5-2} = 3$

$a_n = a_1 + d(n-1)$

$-2 = a_1 + 3(2-1)$

$\boxed{a_1 = -5}$

b) Recursive $\rightarrow$ $\boxed{a_n = a_{n-1} + 3, \; n \geq 2}$

c) Explicit $\rightarrow$ $a_n = a_1 + d(n-1)$

$a_n = -5 + 3(n-1)$

$\boxed{a_n = 3n - 8}$

More tools to remember:

Definition of a Sequence:

A function with domain = Positive Integers, $\mathbb{Z}^+$

<u>Arithmetic</u>

d = common difference

$a_n = a_1 + d(n-1)$

<u>Geometric</u>

r = common ratio

$a_n = a_1 r^{n-1}$

Chapter 10: Infinite Series

10.1 Graphing a Sequence

Writing an Explicit Rule

Limit of a Sequence

Convergence and Divergence

Graphing a Sequence

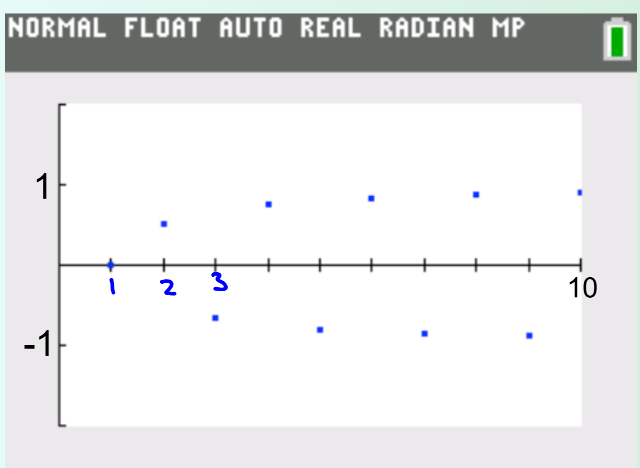
$$a_n = (-1)^n \left(\frac{n-1}{n}\right)$$

MODE

Parametric, dot

T= Integers, T≥1

T step = 1



Writing an Explicit Rule:

Try to recognize patterns

n	1	2	3	4
1)	3,	7,	11,	15, ...

$a_n = 4n - 1$

n	1	2	3	4
2)	1,	$\frac{1}{4}$,	$\frac{1}{9}$,	$\frac{1}{16}$, ...

$a_n = \frac{1}{n^2}$

3) A little harder:

n	1	2	3	4	5
	1,	$\frac{4}{3}$,	2,	$\frac{16}{5}$,	$\frac{16}{3}$, ...

What if I rewrite it like:

$\frac{2}{2}, \frac{4}{3}, \frac{8}{4}, \frac{16}{5}, \frac{32}{6}, \ldots$

$\frac{2^n}{n+1}$ $\frac{2^n}{n+1}$

Limit of a Sequence

If $\lim_{n \to \infty} a_n = L$ L is a Real #

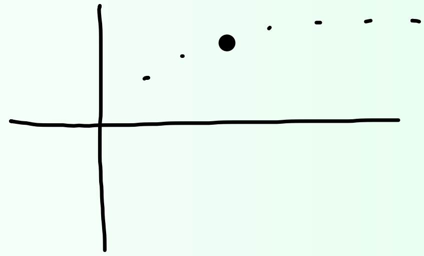
Then the sequence converges.

If the sequence has no real # limit, then it diverges.

* Limit Properties from section 4.5 are restated on p. 561 for sequences.

Graph the sequence to see if it looks like it converges. If it does, confirm by taking the limit.

$$a_n = \frac{2n - 1}{n}$$



$$\lim_{n\to\infty} \left(2 - \frac{1}{n}\right)$$

$$2 - 0$$

$$\boxed{2}$$

Remember this?

$$\lim_{n\to\infty} \frac{n^4 - 3n + 1}{5n^4 - 2} \cdot \frac{\frac{1}{n^4}}{\frac{1}{n^4}}$$

$$\lim_{n\to\infty} \frac{1 - \frac{3}{n^3} + \frac{1}{n^4}}{5 - \frac{2}{n^4}}$$

$$\frac{1}{5}$$

Review: Definition of Factorial

Let n = $\mathbb{Z}^+$ $\quad n! = 1 \cdot 2 \cdot 3 \cdot 4 \ \ldots \ (n-1)n$

$0! = 1$

*Be careful of Order of Operations

$2x^3 \neq (2x)^3$

$2n! \neq (2n)!$

HW:

p. 570, # 1 - 13 odd,

25 - 45 odd